Find $\frac{dy}{dx}$ if $(x^3y^2 - y^3 + 1)^2 = 2 - x^2$. **SCORE: _____ / 6 PTS**

$$\underbrace{2(x^3y^2-y^3+1)}_{1}\left(\underbrace{3x^2y^2}_{\frac{1}{2}}+\underbrace{x^3(2y)\frac{dy}{dx}}_{1}\underbrace{-3y^2\frac{dy}{dx}}_{1}\right)=\underbrace{-2x}\ \frac{1}{2}$$

$$3x^2y^2+(2x^3y-3y^2)\frac{dy}{dx}=-\frac{x}{x^3y^2-y^3+1}$$

$$\frac{dy}{dx}=\boxed{\frac{-\frac{x}{x^3y^2-y^3+1}-3x^2y^2}{2x^3y-3y^2}}\ 1$$

$$=\boxed{\frac{-x-3x^2y^2(x^3y^2-y^3+1)}{(2x^3y-3y^2)(x^3y^2-y^3+1)}}\ 1$$

Prove the derivative of $\arccos x$ using implicit differentiation. **SCORE: _____ / 4 PTS**

$y = \cos^{-1} x$

$\cos y = x$ $\frac{1}{2}$ AND $y \in [0, \pi]$ $\frac{1}{2}$ $\longrightarrow$ $\sin y \geq 0$

$(-\sin y)\frac{dy}{dx} = 1$ $\frac{1}{2}$

$\sin^2 y + \cos^2 y = 1$

SO, $\sin y = \sqrt{1-\cos^2 y} = \sqrt{1-x^2}$ $\frac{1}{2}$

$\frac{dy}{dx} = -\frac{1}{\sin y}$ $\frac{1}{2}$

$= -\frac{1}{\sqrt{1-x^2}}$ $\frac{1}{2}$

$$\frac{d}{dx} \ln \frac{\tan^4 x}{\sqrt[3]{x + \sin x}}$$

1

$$= \underbrace{\frac{d}{dx}\left(4 \ln \tan x - \frac{1}{3}\ln(x+\sin x)\right)}$$

$$= \underbrace{\frac{4}{\tan x}\sec^2 x}_{2} - \underbrace{\frac{1}{3}\frac{1}{x+\sin x}(1+\cos x)}_{2}$$

$$= \frac{4\sec^2 x}{\tan x} - \frac{1+\cos x}{3(x+\sin x)}$$

$$\frac{d}{dy}\arcsin(e^{y^2}+\csc y)$$

$$=\frac{1}{\sqrt{1-(e^{y^2}+\csc y)^2}}\cdot(e^{y^2}\cdot 2y-\csc y\cot y)$$

$$=\frac{\underbrace{2ye^{y^2}}_{②}\underbrace{-\csc y\cot y}_{①}}{\underbrace{\sqrt{1-(e^{y^2}+\csc y)^2}}_{②}}$$

$$\frac{d}{dt}\tan^{-1}\frac{t^2}{1-t}$$

$$= \boxed{\frac{1}{1+\left(\frac{t^2}{1-t}\right)^2}} \cdot \boxed{\frac{2t(1-t)-t^2(-1)}{(1-t)^2}}$$

2 2

$$= \frac{2t-2t^2+t^2}{(1-t)^2+t^4}$$

$$= \boxed{\frac{2t-t^2}{(1-t)^2+t^4}}$$ 1

$$\frac{d}{d\theta}(\sec\theta)^{\theta^2}$$

$$y = (\sec\theta)^{\theta^2}$$

$$\underbrace{\ln y = \theta^2 \ln \sec\theta}\ \frac{1}{2}$$

$$\underbrace{\frac{1}{y}\frac{dy}{d\theta}}_{\frac{1}{2}} = \underbrace{2\theta \ln \sec\theta}_{\frac{1}{2}} + \underbrace{\theta^2 \frac{1}{\sec\theta} \sec\theta \tan\theta}_{2}$$

$$= 2\theta \ln \sec\theta + \theta^2 \tan\theta$$

$$\frac{dy}{d\theta} = y\left(2\theta \ln \sec\theta + \theta^2 \tan\theta\right)$$

$$= \underbrace{(\sec\theta)^{\theta^2}}_{1}\underbrace{\left(2\theta \ln \sec\theta + \theta^2 \tan\theta\right)}_{\frac{1}{2}}$$

SEE ALSO ALTERNATE SOL'N

(GRADE AGAINST ONE VERSION OF SOLUTION ONLY)

ALTERNATE SOLUTION

$$\frac{d}{d\theta}(\sec\theta)^{\theta^2}$$

$$= \frac{d}{d\theta}\left(e^{\ln\sec\theta}\right)^{\theta^2}$$

$$= \frac{d}{d\theta}\underbrace{e^{\theta^2\ln\sec\theta}}_{1}$$

$$= \underbrace{e^{\theta^2\ln\sec\theta}}_{\frac{1}{2}}\Big(\underbrace{2\theta\ln\sec\theta}_{\frac{1}{2}} + \underbrace{\theta^2\frac{1}{\sec\theta}\sec\theta\tan\theta}_{2}\Big)$$

$$= \underbrace{(\sec\theta)^{\theta^2}}_{\frac{1}{2}}\underbrace{\left(2\theta\ln\sec\theta + \theta^2\tan\theta\right)}_{\frac{1}{2}}$$